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TURBULENCE IN A FOLDED PATH

by

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BEAM SPREAD INDUCED BY ATMOSPHERIC TURBULENCE IN A FOLDED PATH

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ABSTRACT: On the basis of extended Huygens-Fresnel principle and quadratic approximation, general expressions of the beam spread reflected from a target in a turbulent atmosphere are derived. The properties of reflected beam spread are investigated. It is found that there exist "amplification" and "self-compensation" of beam spread in the reflection from a mirror and a corner cube reflector, respectively. Calculations show that the spread can be reduced by using a long-wavelength laser as well as large transmitting and reflection radii.

KEY WORDS: atmospheric turbulence, reflected beam spread.

1. Introduction

With the growth of applications in the field of atmosphere, more and more interest has been given to the research on new turbulence effect laws with reference to the reflected beams. In the past few years, large volumes of theoretical and experimental

work was done in such areas as the fluctuation of reflecting field in the turbulent atmosphere, the effect of transmitting and reflection radii on the intensity of the reflection field, the trembling of reflected beam images in the telescope, as well as the amplitude fluctuation time spectrum and space correlation function of the infinite plane wave and spherical wave reflection field.

However, up till now, little has been known about the reflected beam spread. This paper serves in a discussion of this problem.

2. Derivation of Reflected Laser Beam Spread Formulas

In numerous laser engineering projects, a corner reflector is often used as a cooperative target, with which a laser beam emitted by a transmitting telescope to a reflector and reflected back to a receiving telescope, passes through the same turbulent atmosphere twice on its return trip. Let the turbulent atmosphere be a non-loss medium, then based on the extended Huygens-Fresnel principle[1], the incident field at point (L,p) of the corner reflector is:

$$\left.\begin{aligned} u_i(\boldsymbol{p}) &= \frac{k}{2\pi i}\int u_0(\boldsymbol{x})G(\boldsymbol{p},\boldsymbol{x})d^2\boldsymbol{x}, \\ G(\boldsymbol{p},\boldsymbol{x}) &= G_0(\boldsymbol{p},\boldsymbol{x})\exp[\psi_i(\boldsymbol{p},\boldsymbol{x})], \\ G_0(\boldsymbol{p},\boldsymbol{x}) &= [\exp(ik|\boldsymbol{p}-\boldsymbol{x}|)]/|\boldsymbol{p}-\boldsymbol{x}|, \end{aligned}\right\} \qquad (1)$$

where $k=(2\Pi/\lambda)$ is the number of waves; λ is laser wavelength; $u_0(x)$ is the initial field distribution of the transmitting end; $G(p,x)$ is Green's function in the turbulent atmosphere; $G_0(p, x)$ is Green's function in vacuo; $\psi_i(p,x)$ is the turbulence-induced reset phase fluctuation at point (L,p) of the spherical wave emitted at point (0,x). Following near-axis approximation[2], Eq. (1) can be written as

$$u_i(\boldsymbol{p}) = -\frac{ik}{2\pi L}\exp(ikL)\int d^2\boldsymbol{x}\,u_0(\boldsymbol{x})\exp\left[\frac{ik}{2L}|\boldsymbol{p}-\boldsymbol{x}|^2+\psi_i(\boldsymbol{p},\boldsymbol{x})\right]. \qquad (2)$$

The reflection field from the corner reflector can satisfy[3]

$$u_r(p) = r(p)u_i^*(p), \tag{3}$$

where r(P) is the effective amplitude reflection coefficient of the corner reflector; u_i^* is the composite conjugation of the incident field. Again, by applying the extended Huygens-Fresnel principle to the reflection field, the reflection field at point (0,y) of the receiver can be derived as

$$u_r(y) = -\frac{ik}{2\pi L}\exp(ikL)\int d^2p\, r(p)u_i^*(p)\exp\left[\frac{ik}{2L}|y-p|^2+\psi_r(y,p)\right], \tag{4}$$

where ψ_r(y,p) is the turbulence-induced reset phase fluctuation at point (0,y) of the spherical wave at point (L,p). By inserting the composite conjugation of Eq. (1) in Eq. (4), the following can be obtained:

$$\begin{aligned} u_r(y) = \left(\frac{ik}{2\pi L}\right)^2 \int d^2x\, d^2p\, u_0^*(x)r(p)\exp\left[\frac{ik}{2L}(|y-p|^2-|p-x|_2)\right] \\ \times\exp[\psi_r(y,p)+\psi_i^*(p,x)]. \end{aligned} \tag{5}$$

From here, the mutual correlation function of the reflection field can be calculated as

$$\begin{aligned} \Gamma_2(y_1,y_2) &= \langle u_r(y_1)u_r^*(y_2)\rangle \\ &= \left(\frac{k}{2\pi L}\right)^4 \int d^2x_1 d^2x_2 d^2p_1 d^2p_2 u_0^*(x_1)u_0(x_2)r(p_1)r^*(p_2) \\ &\quad\times\exp\left[\frac{ik}{2L}(|y_1-p_1|^2-|p_1-x_1|^2-|y_2-p_2|^2+|p_2-x_2|^2)\right] \\ &\quad\times\langle\exp[\psi_r(y_1,p_1)+\psi_r^*(y_2,p_2)+\psi_i(p_2,x_2)+\psi_i^*(p_1,x_1)]\rangle. \end{aligned} \tag{6}$$

The average light intensity is

$$\begin{aligned} \langle I(y)\rangle = \left(\frac{k}{2\pi L}\right)^4 \int d^2x_1 d^2x_2 d^2p_1 d^2p_2 u_0^*(x_1)u_0(x_2)r(p_1)r(p_2)M_L \\ \times\exp\left[\frac{ik}{2L}(|y-p_1|^2-|p_1-x_1|^2-|y-p_2|^2+|p_2-x_2|^2)\right], \end{aligned} \tag{7}$$

where M_L is double-source spherical wave coherence function. Assume the light propagation is suitable for the "Markov approximation", then

$$M_L = \langle\exp[\psi_r(y,p_1)+\psi_r^*(y,p_2)]\rangle\langle\exp[\psi_i(p_2,x_2)+\psi_i^*(p_1,x_1)]\rangle. \tag{8}$$

If the points (L,0) and (L,P_j) are located in the same isoplanatic domain[4], then the optical wave will pass through the same turbulence while traveling from the point (0,x) to the point (L,P_j) or the point (L,0). In this case, the dependance of ψ_i on P_j can be ignored, i.e., the following can be established:

$$\psi_i(p_j, x_j) \simeq \psi_i(0, x_j), \quad (j=1, 2) \tag{9}$$

Generally speaking, the scale of the isoplanatic domain is approximately the order of magnitude of either the outer scale of turbulence L_0 or the coherence length of the spherical wave ρ_0, which has a smaller value. Here, the coherence length of the spherical wave is

$$\rho_0 = (0.546 C_n^2 k L)^{-3/5}, \tag{10}$$

where C^2_n is refractivity structure constant. Hence, Eq. (8) can be rewritten as

$$M_L = \langle \exp[\psi_r(y, p_1) + \psi_r^*(y, p_2)] \rangle \langle \exp[\psi_i(0, x_2) + \psi_i^*(0, x_1)] \rangle. \tag{11}$$

Let the turbulence be stable statistically, and the reset phase fluctuation be a Gaussian random quantity, then [5]

$$M_L = \exp\left\{-\frac{1}{2}[D_\psi(0, p_1 - p_2) + D_\psi(0, x_1 - x_2)]\right\}, \tag{12}$$

where $D_\psi(\Theta,\Theta') = D_\chi(\Theta,\Theta') + D_s(\Theta,\Theta')$ is the structure function of the double-source spherical wave in turbulent atmosphere; $D_\chi(\Theta,\Theta')$ and $D_s(\Theta,\Theta')$, respectively, are logarithmic amplitude and phase structure function. For a local uniform and isotropic turbulence, the wave structure function is[6]

$$D_z(\theta, \theta') = 2.91 k^2 L \int_0^1 d\eta C_n^2(\eta L) [|\theta\eta + \theta'(1-\eta)|^{5/3}]. \tag{13}$$

Generally speaking, Eq. (13) can hardly get an analytical solution, and therefore requires some kinds of approximation, of which, a better approximation is "quadratic approximation"[7,8] that leads to

$$D_\psi(\theta, \theta') = 2\rho_0^{-2}(\theta^2 + \theta'^2 + \theta\cdot\theta'), \tag{14}$$

Thus,

$$M_L = \exp[-\rho_0^{-2}(|p_1 - p_2|^2 + |x_1 - x_2|^2)]. \tag{15}$$

In addition, let both the initial transmitting field and the

effective amplitude reflection coefficient of the corner reflector contain Gaussian distribution, i.e.

$$u_0(x)=u_0\exp[(-1/2)k\alpha x^2],\quad r(p)=r_0\exp(-p^2/2b^2), \tag{16}$$

where $\alpha=\alpha_1+\alpha_2=(1/k\alpha^2)+(i/F)$; α and b, respectively, are transmitting aperture and the effective radius of the reflector; F is the curvature radius of transmission beam. By introducing Eq. (15) in the integration formula (7), the following can be derived through operation:

$$\langle I(y)\rangle=\frac{1}{32}\left(\frac{k}{L}\right)^4 u_0^2 r_0^2\,\frac{\exp(-wy^2)}{mnth}, \tag{17}$$

$$\left.\begin{aligned}
m&=\frac{1}{2}k\alpha_1+\rho_0^{-2}+i\left(\frac{1}{2}k\alpha_2-\frac{k}{2L}\right),\\
n&=\frac{1}{2}k\alpha_1+\frac{ik}{2L}+\rho_0^{-2}-\frac{4\rho_0^{-4}}{m},\\
t&=\frac{1}{2b^2}+\rho_0^{-2}+\frac{k^2}{m^2L^2}+\frac{16k^2\rho_0^{-4}}{m^2nL^2},\\
h&=\frac{1}{2b^2}+\rho_0^{-2}+\frac{k^2}{nL^2}-\frac{A}{t},\\
A&=2\rho_0^{-2}+\frac{8k^2\rho_0^{-2}}{mnL^2},\\
w&=\frac{k^2}{L^2t}-\left(\frac{k}{L}-\frac{2k}{Lt}A\right)^2h^{-1}.
\end{aligned}\right\} \tag{18}$$

Beam spread can be expressed with mean square radius $\langle\rho_L^2\rangle$, which can be defined as

$$\langle\rho_L^2\rangle=\int\rho^2\langle I(\rho)\rangle d^2\rho\Big/\int\langle I(\rho)\rangle d^2\rho, \tag{19}$$

By inserting Eq. (17) in Eq. (19), the expression for the mean-square radius of the reflected beam from the corner reflector can be acquired as follows:

$$\begin{aligned}
\langle\rho_L^2\rangle_c=\frac{8L^4}{k^4}\Bigg\{&\left[\frac{1}{2b^2}+\rho_0^{-2}+\frac{dk^2}{4L^2(d^2+c^2-\rho_0^{-4})}\right]^2+\left[\frac{ck^2}{4L^2(d^2+c^2-\rho_0^{-4})}\right]^2\\
&-\left[\rho_0^{-2}+\frac{k^2\rho_0^{-2}}{4L^2(d^2+c^2-\rho_0^{-4})}\right]^2\Bigg\}\left[\frac{2L^2}{b^2k^2}+\frac{d-\rho_0^{-2}}{d^2+c^2-\rho_0^{-4}}\right]^{-1}.
\end{aligned} \tag{20}$$

Similarly, in the case of the plane mirror (r(P)=constant)

$$\langle\rho_L^2\rangle_m = \frac{8L^4}{k^4}\left\{\left[\rho_0^{-2}+\frac{dk^2}{4L^2(d^2+c^2-\rho_0^{-4})}\right]^2+\left[\frac{k}{L}-\frac{ck^2}{4L^2(d^2+c^2-\rho_0^{-4})}\right]^2 - \left[\rho_0^{-2}+\frac{k^2\rho_0^{-2}}{4L^2(d^2+c^2-\rho_0^{-4})}\right]^2\right\}\left(\frac{d-\rho_0^{-2}}{d^2+c^2-\rho_0^{-4}}\right)^{-1}, \tag{21}$$

where

$$d=(2a^2)^{-1}+\rho_0^{-2}, \quad c=(k/2F)-(k/2L).$$

3. Reflected Beam Spread Properties

To simplify, we introduced the concept of the reflected beam spread amplification coefficient, which can be defined as a ratio between the effective radii of the beams at the reflection path and at the distance double the single trip, i.e.,

$$G=(\langle\rho_L^2\rangle_{c,m})^{1/2}/(\langle\rho_{2L}^2\rangle_s)^{1/2}, \tag{22}$$

where $\langle\rho_L^2\rangle_{c,m}$ can be derived from either Eq. (20) or (21), while $\langle\rho_{2L}^2\rangle_s$ is determined by the following formula[9]:

$$\langle\rho_{2L}^2\rangle_s = a^2\left[\left(-\frac{2L}{F}\right)^2+4\left(1+\frac{4a^2}{\rho_0^2}\right)L^2k^{-2}a^{-4}\right]. \tag{23}$$

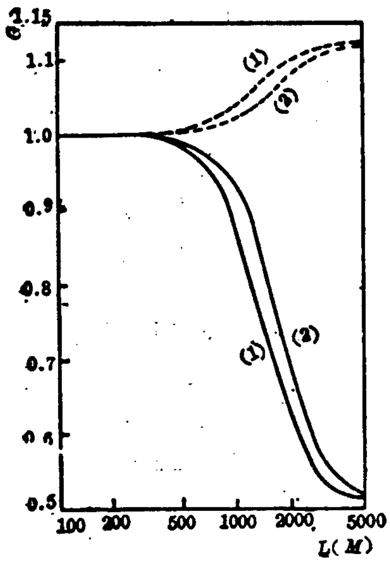

Fig. 1 Amplification coefficient G vs field range L for different wavelength

$C_n^2 = 10^{-13}\, m^{-2/3}$, $a = 10$ cm, $b = 20$ cm

—— corner cube reflector reflection,

- - - - - mirror reflection

1—$\lambda = 0.6328\ \mu m$; 2—$\lambda = 3.8\ \mu m$

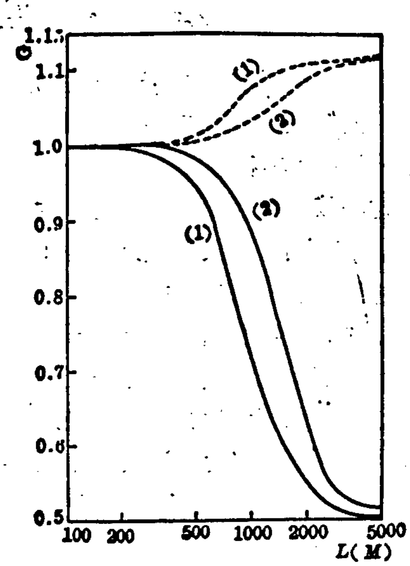

Fig. 2 Amplification coefficient G vs field range L for different aperture size

$C_n^2 = 10^{-13}\, m^{-2/3}$, $\lambda = 1.06\ \mu m$

—— corner cube reflector reflection,

- - - - - mirror reflection

1—$a = 5$ cm, $b = 10$ cm; 2—$a = 10$ cm, b = 20 cm

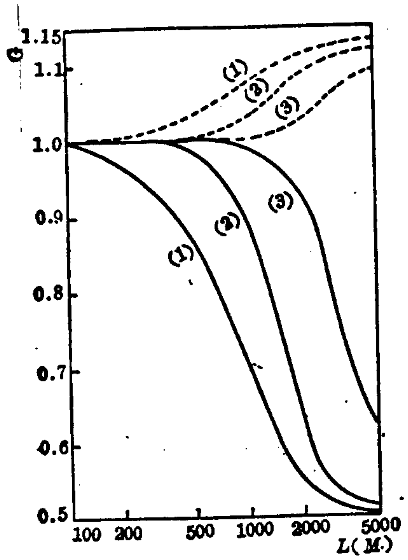

Fig. 3 Amplification coefficient G vs field range L for different turbulence intencity
$\lambda=1.06\,\mu m$, $a=10$ cm, $b=20$ cm
—— corner cube reflector reflection,
- - - - - mirror reflection
1—$C_n^2=10^{-12}\,m^{-2/3}$; 2—$C_n^2=10^{-13}\,m^{-2/3}$;
3—$C_n^2=10^{-14}\,m^{-2/3}$

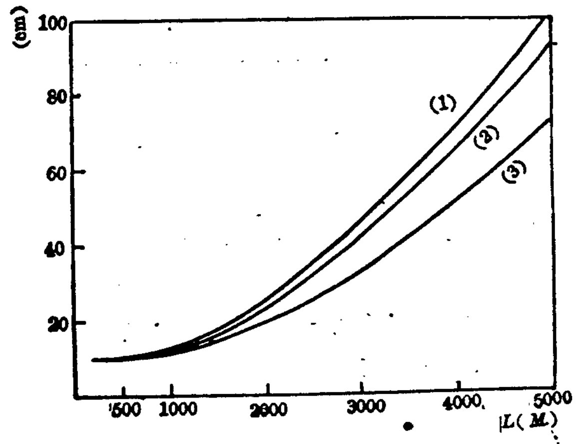

Fig. 4 Effective radius r_m vs field range L for different wavelength
$C_n^2=10^{-13}\,m^{-2/3}$, $a=10$ cm
1—$\lambda=0.6328\,\mu m$; 2—$\lambda=1.06\,\mu m$;
3—$\lambda=3.8\,\mu m$

Figs. 1-3 show calculations of the reflected focused beam (F=2L) spread amplification times under different parameters. It can be seen from these figures that the amplification coefficient of the plane mirror reflected beam spread is G>1, i.e.. the effective radius of the plane mirror reflected beam $r_m=(\langle\rho^2_L\rangle_m)^{1/2}$ is greater than the effective radius of the beam propagating over the same distance on a single trip $r_s=(\langle\rho^2_{2L}\rangle_s)^{1/2}$, which is the so-called amplification effect. This amplification effect is believed to be caused by the coherence between the reflection field and incident field[9]. Over a shorter distance (L<500m), this effect is extremely small, and the amplification coefficient is approximately 1, while over a longer distance, the amplification effect is saturated. On the whole, however, reflection from the plane mirror

has no much difference from direct propagation (G≤1.1).

The result is reverse in the case of the corner reflector, i.e., the effective radius of the reflected beam $r_c=(\langle\rho^2_{2L}\rangle_c)^{1/2}$ is smaller than the effective radius during single trip propagation. This phenomenon is referred to as self-compensation effect. This is because the corner reflector can make the reflected light return back along the original incidence direction, which will cause a decrease in geometric divergence, as well as in the fluctuation of beam arrival angle at the receiving end and the beam drift; as a result, beam spread can be "compensated".

This effect was confirmed by Kerr's experiment[10]. The degree of self-compensation is related to various parameters, of which, the most important are propagation distance and turbulence intensity. In the best case, the scale of the reflected beam from the corner reflector can be improved by around one time compared with that during direct propagation G is approximately equal to 0.5.

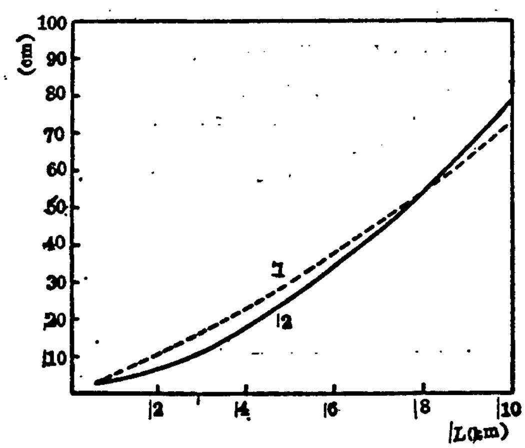

Fig. 5 Effective radius r_m vs field range L for different wavelength
$C_n^2=10^{-14}\,m^{-2/3}$, $a=2.5$cm
1—$\lambda=3.8\,\mu$m; 2—$\lambda=1.06\,\mu$m

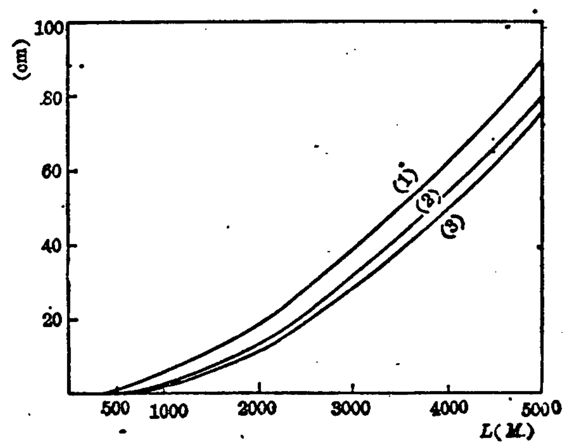

Fig. 6 Relative effective radius r_m vs field range L for different aperture size
$C_n^2=10^{-13}\,m^{-2/3}$, $\lambda=1.06\,\mu$m
1—$a=2.5$ cm; 2—$a=10$ cm; 3—$a=15$ cm

Figs. 4-6 show the effect of wavelength and transmitting aperture on the effective radius r_a in the case of the plane mirror reflected focused beam. It can be seen from the figures that the effect of wavelength and transmitting aperture on the effective radius conforms to Eq. (23) which describes the beam spread during direct propagation.

4. Conclusions

To summarize, a theoretical analysis was made of the turbulent atmosphere reflected laser beam spread. The major results the authors obtained include:

(1) When a plane mirror is used as the reflector, the reflected beam spread displays an amplification effect, while with a corner reflector, the reflected beam spread shows a self-compensation effect.

(2) The reflected beam spread can be reduced using a large transmission and reflection radii as well as a long wavelength laser.

In the process of derivation, we applied "quadratic approximation" and demonstrated that this approximation is reasonable in ordinary cases[11]. In addition, the Markov approximation is applicable to almost all optical wave propagation processes. Nevertheless, our results proved suitable only to the weak and medium intensity fluctuations with "isoplanatic domain" approximation.

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